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Comparison of Two Mathematical Models for the Surface Reconstruction for Deformation Analysis by Using FARO Focus 3D

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Background

In real world, many disasters are close related to deformation, such as earthquake, landslide, and collapse of dams and bridges. Thus, deformation monitoring raises a lot of attention worldwide. Deformation monitoring or deformation survey is the systematic measurement and tracking of the alteration in the shape or dimensions of an object as a result of stresses induced by applied loads. With the rapid development of technology and the requirement of deformation monitoring, the deformation monitoring technology is also continuously improving. The application of 3D laser scanning provides a brand-new technology for human beings to obtain abundant spatial information. The technique is developed from manual single point measurement to automated multi-point data collection, so that the measurement efficiency is substantially improved. No mater GPS, total station or other traditional deformation monitoring methods have a common ground, they can measure the same points in different epochs, the methods which mentioned in the thesis except laser scanning can get together all different times data and do the statistics also easily apply to adjustment calculation, predicting the point where deformation will be occurred; On the contrary, laser scanner deformation monitoring has no this characteristic, it can’t surely measure the same points on the object at different epochs, which implied that there is no statistical information to predict the occur of deformation for an object.

Objective

The purpose of this thesis is to compare two mathematical models for the surface reconstruction for deformation analysis by using FARO Focus 3D, trying to find a proper model, which can reconstruct the surface properly and predict the deformation not only in points but in element surfaces. We utilize the advantages of 3D laser scanner, fast area wide data acquisition, scanning the object-bridge support model, which is available in the K1 measurement basement and the tachymeter (TS 30) is used to control the measurements. The laser scanner we used is
FARO 3D Focus scanner. The resulting data is then applied to Non Uniform Rational B-Splines (NURBS) and triangulation mathematical models, and reconstructing the surface of the beam, then we compare the differences between the two methods.

**Methodology of Surface Reconstruction**

**NURBS**

NURBS (Non-Uniform Rational Basis Spline) is a mathematical model which often used in computer graphic and for generating and representing curves and surfaces, which offers great flexibility and precision for handling both analytic and modeled shapes.

Basis function used in NURBS:

\[ N_{i,n}(u) = f_{i,n} N_{i,n-1} + g_{i+1,n} N_{i+1,n-1} \]

Where

- \( N_{i,n}(u) \) = basis function
- \( f_{i,n} \) and \( g_{i,n} \) = weight function

![Diagram of NURBS basis functions](image)

*Figure 1. A curve is constructed through the control points*
A general form of a NURBS curve [Les Piegl & Wayne Tiller]:

\[ C(u) = \sum_{i=1}^{k} \frac{N_{i,n}(u) \omega_i}{\sum_{j=1}^{k} N_{j,m}(v) \omega_j} P_i = \frac{\sum_{i=1}^{k} N_{i,n}(u) \omega_i P_i}{\sum_{i=1}^{k} N_{i,n}(u) \omega_i} \]

Where

\( k \) = Number of control points of \( P_i \)

\( \omega_i \) = Corresponding weights

A NURBS surface is obtained as the tensor product of two NURBS curves, thus using two independent parameters \( u \) and \( v \) (with indices \( i \) and \( j \) respectively) [Les Piegl & Wayne Tiller]:

\[ S(u, v) = \sum_{i=1}^{k} \sum_{j=1}^{l} R_{i,j}(u, v)P_{i,j} \]

With Rational Function:

\[ R_{i,j}(u, v) = \frac{N_{i,n}(u)N_{j,m}(v)\omega_{i,j}}{\sum_{p=1}^{k} \sum_{q=1}^{l} N_{p,n}(u)N_{q,m}(v)\omega_{p,q}} \]

Where

\( N_{i,n}(u) \) = basis function

\( i = i \)– th control points

\( \omega = \) corresponding weights
Delaunay Triangulation

Delaunay triangulations maximize the minimum angle of all the angles of the triangles in the triangulation; they tend to avoid skinny triangles. For the set of points on the same line, no Delaunay triangulation (the concept of triangulation is degraded in this case). Four or more points on a circle (for example, the vertex of the rectangle) Delaunay triangulation is not unique each quadrilateral into two triangles are two possible triangulation to meet the Delaunay condition, requiring the internal space of all the circumcircle of the triangle.

![Figure 2. Showing a Delaunay triangulation in the plane with circumcircles](image)

![Figure 3. Delaunay Condition](image)

\[
\begin{vmatrix}
A_x & A_y & A_x^2 + A_y^2 & 1 \\
B_x & B_y & B_x^2 + B_y^2 & 1 \\
C_x & C_y & C_x^2 + C_y^2 & 1 \\
D_x & D_y & D_x^2 + D_y^2 & 1 \\
\end{vmatrix} = \begin{vmatrix}
A_x - D_x & A_y - D_y & (A_x^2 - D_x^2) + (A_y^2 - D_y^2) \\
B_x - D_x & B_y - D_y & (B_x^2 - D_x^2) + (B_y^2 - D_y^2) \\
C_x - D_x & C_y - D_y & (C_x^2 - D_x^2) + (C_y^2 - D_y^2) \\
\end{vmatrix} > 0
\]
**Data Acquisition and Analysis**

**Flow Chart for Measurement Concept**

- **Free Stationing**
- **Tachymeter Measurement:** Measure five control points
- **FARO Scanner Before Deformation:** Scan targets on the wall and small targets on object control points
- **Deformation**
- **Same measurement procedure as before deformation**
- **Data Processing**

**Flow Chart for Data Processing**

- **Collected Data importing to FARO Scene**
- **Registration**
- **Record the coordinates of targets**
- **Test reliability of data**
- **Comparison of differences between tachymeter and FARO Focus scanner**

**Results**

**NURBS in Rhinoceros**

- **Import Data**
- **Projection in Matlab**
- **Mesh the Points Cloud**
- **Apply to NURBS**
- **Export Data**

*Figure 4. Procedure in Rhinoceros Software*
The first thing we did in Rhinoceros software is that importing data, the next step was to mesh the points cloud and applied to NURBS model; however, we found the result is bad and far away from our expectation (See Figure 5), the details will be explain later. To improve this bad result, we tried to pre-processing the data, which projected all points on a proper surface in Matlab (See Figure 6), then importing the data again into Rhinoceros software to do the same steps to apply to the NURBS model and reconstruct the surface.

The grids are very ordered so the result seems good and clear (See Figure 7), separating two data from different colors, it’s easier to see the deformation, and we can observe that the middle of the object beams overlap between before and after deformation, it’s because of the calculation of grids are not just fit the shape of object beams.
The Figure 13 Shows the surface reconstructed result, which just simulated the surface of the scanned part of the object beam, also showing the final result in Meshlab, it’s a successful result. The two points cloud data can be obviously observed the difference between before deformation and after deformation. We will compare the differences of TIN from before and after deformation.
Comparison of Differences in ArcGIS

Figure 9. Points Data transfer from .XYZ format to Point shape file (Left); TIN made by Point shape file (Right)

Figure 1. Deformation Tilt

The model of before deformation is as the reference, then importing the after deformation data to do the comparison, which compares the difference of the TIN from two models. The method which used in performing geometric comparison the difference is constrained Delaunay triangulation. The areas where the first surface is above the second (reference) surface shows the positive value; and negative values reflect areas where the first surface is below the second (See Figure 11).

Figure 11. At the One End of Object Beam
Figure 12. Reference for Surface

The result of the comparison of difference is the volumetric difference comes from two surfaces comparison, which has a reference (see Figure 12) and each surface has a volume with this reference, the final result is the difference of the two volumes.

Conclusion

The focus of our research was to compare two mathematical models for surface reconstruction, through the results of two mathematical models applied to reconstruct the surfaces, both of the mathematical models were operable; however, we can observe that the triangulation method is better than the NURBS here in our case.

In the future, the first thing we want to improve is that to find the better filter to choose more completely the target object we want. The second thing we can go deep into find the proper software to numerate the result from Rhinoceros; not only can visualize the differences between before and after deformation but also can see the numerical values. The third one is looking forward to improving and exploring more the function of FARO Scene software, which can scan and filter the original data more efficiently.

Using 3D laser scanning in modern world is a trend in deformation monitoring because it can approach 1mm accuracy and obtain more information from object which can get better quality. In our thesis, we tried two mathematical models, triangulation and NURBS, to do the surface reconstruction and compare the results. The result from triangulation model was the better one, we expect that we can polish the method to enhance the result and also find that if there has a mathematical model which can deal with the data and reconstruct better!